REMARKS ON THE PAPER BY L.R. FOWELL, "EXACT AND APPROXIMATE SOLUTIONS FOR THE SUPERSONIC DELTA WING"

(ZAMECHANIE K STAT'E L.R. FAVELLA "TOCHNOE I PRIBLIZHENNOE RESHENIIA DLIA SVERKHZVUKOVOGO DEL'TAOBRAZNOGO KRYLA")

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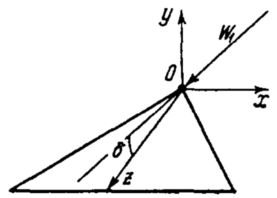
The extensive and important work by L. Fowell [1] contains a solution to the problem of inviscid supersonic flow about a flat delta wing with supersonic leading edges at a finite angle of attack without yaw.



Since the leading edges are supersonic, the conical flows formed by the "top" and the "bottom"* of the wing do not interfere with each other, and therefore can be examined separately. Unfortunately in this interesting work, as it will be shown later, the statement of the boundary conditions for the expansion surface is incorrect. This weakens Fowell's conclusion concerning flow about the expansion surface. The correct picture of the flow and the statement of boundary conditions for a delta wing with supersonic leading edges are contained in the author's paper [2].

If a conical flow has a velocity potential ϕ, then the velocity components along the system of Cartesian coordinates o, x, y, z can be written:

* "Top" and "bottom" stand for the expansion and the compression surfaces of the wing, respectively. (Translator)

$$u = \varphi_x = F_\xi, \quad v = \varphi_y = F_\eta, \quad w = \varphi_z = F - \xi u - \eta v, \quad \varphi = zF(\xi, \eta)$$
$$\left(\xi = \frac{x}{z}, \ \eta = \frac{y}{z}\right)$$

where F satisfies the equation

$$AF_{\xi\xi} + 2BF_{\xi\eta} + CF_{\eta\eta} = 0 \tag{1}$$

Here

$$A = a^2(1+\xi^2) - (u - \xi w)^2, \qquad C = a^2(1+\eta^2) - (v - \eta w)^2$$

$$B = (a^2 - w^2)\xi\eta + (u\eta + v\xi)w - uv, \qquad a^2 = a_1^2 - \frac{\varkappa - 1}{2}(u^2 + v^2 + w^2 - W_1^2)$$

a is the speed of sound, $a_1 W_1$ are, respectively, the speed of sound and the speed at a certain point in the flow, κ is the ratio of specific heats. The plane $\xi\eta$ has a single physical sense; it is the plane $z = 1$ in the space xyz, while ξ, η, respectively, are coordinates of the points x and y in this plane.

$$\Delta = AC - B^2 = a^2[a^2(1 + \xi^2 + \eta^2) - (u - \xi w)^2 - (v - \eta w)^2 - (\xi v - \eta u)^2] \tag{2}$$

Let us examine the picture of the flow about a delta wing in the plane $\xi\eta$ (Fig.2); due to symmetry only one half of the flow is shown, for $\xi > 0$. The representation of the flow according to Fowell is shown in Fig. 2 *a*, and that by the author [2] is given in *b*.

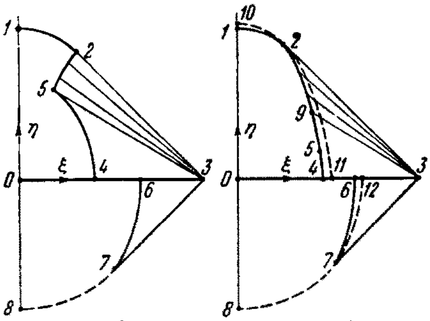


Fig. 2.

The wing is represented by the segment $O3$ (the axis Oz lies in the plane of the wing). The envelope of the Mach cones in the undisturbed flow with apexes at the leading edges appears as the segment 1-2 with apex at the nose of the wing (point 0 in Fig.1). The flow about the sharp leading edge results in an oblique Prandtl-Meyer expansion which continues until the velocity vector is parallel to the wing. This flow is represented by the bundle of straight line characteristics of equation (1) converging at point 3. Segments 3-5 in Fig. 2a and 3-9 in Fig. 2b represent the boundary of the Prandtl-Meyer flow, which is followed by uniform flow, next to the wing surface.

As boundary for the region of the ordinary conical flow for equation (1), Fowell proposes the parabolic line 1-2-5-4 in Fig. 2a, constructed for the part of the conical flow already found. For the undisturbed flow this is part of an arc of the Mach cone 1-2; for the uniform flow adjacer to the surface of the wing it is also part of the Mach cone 5-4; and for the Prandtl-Meyer flow it is the parabolic line 2-5. However, between the Mach cones and the parabolic line 2-5 there is an essential distinction. A Mach cone for uniform flow is at the same time a parabolic line and a characteristic curve of equation (1), since all straight characteristics in a uniform flow touch the Mach cone (see, for example, Ref. [2]), whereas the parabolic line 2-5 for the Prandtl-Meyer flow cannot be a characteristic, for it would then be an envelope of straight characteristics, which is impossible. If it is required that the velocity components of the ordinary conical flow change continuously along 2-5 into the velocity components of a Prandtl-Meyer flow, this is equivalent to the requirement that the function F and its normal derivative be given on 2-5 But the assignment of F and its normal derivative on the non-characteristic curve 2-5 determines, according to the Cauchy-Kovaleskii theorem, a unique analytical solution of equation (1), in a neighborhood of 2-5, and this is the Prandtl-Meyer solution (the curve 2-5 and initial data on it are specified by analytic functions). It follows that it is not possib to join these two different solutions of equation (1) along 2-5 in Fig. 2

For this reason, in [6] the ordinary conical flow in Fig. 2b is separated by the Prandtl-Meyer flow characteristic 2-9, then by the portion of straight characteristic 9-5 passing into a portion of the Mach cone 5-4. Straight characteristics cannot be extended to the surface of the wing, because, as the angle of attack δ decreases, the region of the ordinary conical flow would fill all the interior space 1-2-3-0. Since the characteristic 9-5 is straight, the adjacent flow must be a simple wave [3].

Properties of simple waves were investigated by the author in [2]. It was shown that for motion along curved characteristics of a simple wave, passing through 9-5, the encountered parabolic points cannot form a continuous parabolic line, along which the simple wave could be joined to a solution of elliptic type, which will exist in the interior portion of the ordinary conical flow (at the point $O\Delta > 0$). This conclusion is correct if the simple wave has piecewise continuous third derivatives of F in the vicinity of the parabolic line.

In Ref. [2], the case in which the curved characteristics of a simple wave converge to one parabolic point was not investigated. Assuming that F is sufficiently smooth, it can be shown that if the derivative of acceleration in a direction normal to the straight characteristic, upon which this parabolic point is located, is different from zero (the acceleration must become zero [2]), then the characteristics cannot converge to one point.

All these singularities of simple waves and certain other general considerations of boundary problems connected with equations of mixed type led the author to a conclusion about the formation of a shock wave, which emerges from the parabolic point 2, where its intensity is zero, and lies in the vicinity of the curved characteristic 2-9, straight characteristic 9-5, and the segment of the Mach cone 5-4 (dotted line 2-11 in Fig. 2b). The segment of the Mach cone can serve as a boundary of the conical flow; however if this is adjacent to a uniform flow, there must be a special, particular structure in the vicinity of the Mach cone [4]. For this reason the author considers the possible formation of a weak shock 2-10 and includes it in the formulation of the boundary conditions, because, if such a discontinuity does not exist, we automatically obtain the segment of the Mach cone 1-2, in the solution of the boundary-value problem. Within region 10-2-11-0 in Fig. 2b the author assumes that $\Delta = (AC - B^2) > 0$. The boundary value problem is formulated in Ref. [2].

Fowell presents experimental data, which corroborates the scheme of the author of the present paper. According to Fowell's scheme, there are two regimes of flow on the expansion surface of the wing: without a lateral shockwave and with a shockwave. The first case is observed when point 5 on Fig. 2a is not on line 1-0; when it is on line 1-0 a discontinuity is formed. The angle of attack at which this occurs, Fowell called a critical one. In the neighborhood of the critical angle of attack there would have to be a sharp change in pressure distribution at the wing. The experiments indicated that such a change in pressure distribution does not take place, but instead, the lateral shockwave 2-11 in Fig. 2b is formed at small angles of attack, which completely corroborates the validity of the author's scheme.

Fowell's concept of flow on the compression surface of the wing coincides with the one presented in Ref. [2]. On the leading edge a plane shock 3-7 is formed (Fig. 2) followed by uniform flow. The region of the ordinary conical flow is bounded by the curved shock 7-8 and by the segment of the Mach cone for the uniform flow following the shockwave 6-7. Here the author also introduced a possible discontinuity 7-12 in Fig. 2b.

Fowell examined the rotational conical flow formed on the compression surface of the wing and came to the conclusion that the constant entropy lines converge at the point 0 (Fig.2) where Ferri's vortex singularity occurs. This appears to be a particular case in the general pattern of conical flows.

Let us examine the path of a gas particle moving in a conical flow and its projection on the plane ξ, η; the equation of these lines, which we shall call flow lines, will be

$$\frac{d\xi}{u - \xi w} = \frac{d\eta}{v - \eta w}$$

Along these lines the entropy remains constant. It can be shown that for a uniform flow about any conical body there are two cases. The first case exists when the flow lines run into the surface of the body, which appears in this case as an isobar; the second case occurs when the flow lines converge into one or several points, where Ferri singularities are formed. For instance, in a flow around an elliptic cone at zero angle of attack, two Ferri singularities are formed (Fig.3).

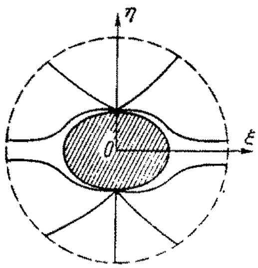


Fig. 3.

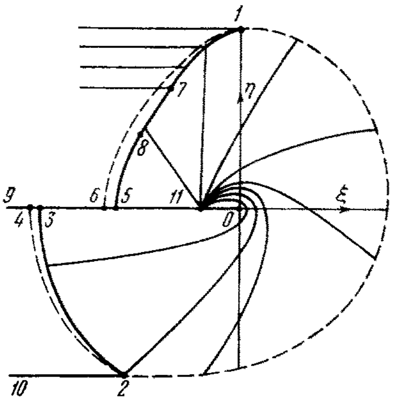


Fig. 4.

In a flow about an edge of a rectangular plate, only one Ferri singularity is formed (Fig.4). Here 0-9 is the wing; 1-7 is a characteristic of the Prandtl-Meyer flow formed at the leading edge; 7-8, 5-8 are a straight characteristic and the Mach cone for the uniform flow; 1-6 is a shockwave forming a boundary of the ordinary conical flow on the upper part of the wing; 1-2 is a shock wave; point 2 is the intersection of the Mach cone of the flow behind the plane shock 10-2 from the leading edge with the shock line 10-2; 2-4 is a possible discontinuity. All the lines converge to point 11 located on the upper surface. It should be noted that Lighthill [5] was the first to deduce the formation of a shock wave 1-6. This was based on the behavior of the linearized solution in the vicinity of the Mach cone, which, as mentioned in Ref. [6] appears to be incorrect. For this reason Lighthill's deduction concerning the formation of the shock wave 1-6 (Fig.2) is not sufficiently convincing.

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